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Current characteristics of electrically sprayed glycerine

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Department:

CURRENT CHARACTERISTICS
OF
ELECTRICALLY SPRAYED GLYCERINE

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ABSTRACT

The current carried by electrically-sprayed glycerine was measured, and an empirical relation was obtained for single-jet spraying, given by

$$I = K_I V^{0.45} \dot{M}^{0.55} d^{-0.18}$$

where V is the electrode voltage, $\dot{M}$ the mass-flow rate, and d the electrode spacing, the electrodes being a capillary and collecting plate. Peculiarities of the current waveform, including an exponentially rising envelope, for unstable-jet spraying and multiple-jet spraying, were recorded.

ACKNOWLEDGEMENTS

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I. DISCUSSION OF PROBLEM

In 1915 and 1917 Zeleny[1,2] reported the phenomenon of electrical or electrostatic spraying of liquids. Since that time liquid spraying has been studied with many purposes in mind such as space propulsion[3,4] and atomization techniques,[5] especially for spray painting purposes.[6] In these studies the chief concern has been charge-to-mass ratios of individual particles, particle size, and configuration of the spray. In a recent paper DeShon and Carson[7] made mention of what appeared to be a "constant current" effect as the spacing between electrodes was increased. This paper will extend these observations, dealing with the electric current carried by the liquid jet as a function of liquid mass-flow rate, terminal voltage, and electrode spacing.

Electrical spraying of a liquid occurs when a liquid emerging from a capillary is subjected to a stress caused by a potential difference between the capillary and a collecting plate. The electrostatic pressure at the liquid surface is caused by free charge accumulation at the surface and a dielectric stress. When this electrostatic pressure becomes larger than the stabilizing pressure due to surface tension, the charged droplet is accelerated toward the plate.[8] With increasing electric field, the droplet size decreases, and droplet frequency increases.[9] At some critical field value a single liquid thread or jet is formed. If the capillary-to-plate spacing is large, the jet may disintegrate into extremely small droplets as shown in Fig. 1. Further increase in the field results in the formation of multiple jets, some in a steady-state condition, others pulsating. Figure 2 shows photographs of these various spraying modes. In Fig. 2 (c) many of what appear to be continuous jets are actually pulsating. The three modes of spraying described above will be referred to as pulse spraying, single-jet spraying, and multiple-jet spraying.

The current for pulse spraying has been shown to consist of pulses corresponding to the liquid pulses with magnitudes depending upon the charge-to-mass ratio of the droplets. The charge-to-mass ratio of this mode has been the subject of studies by Hendricks[3] and will not be discussed here.

The current carried by the single jet is non-time-varying,[7] a function of electrode voltage, the mass-flow rate, and the electrode spacing. An equation developed by Hines[10] relates the charge per unit mass, q, for a jet to the mass-flow rate, $\dot{M}$, and electric field, E. Hines, using the approximation that the electric field and velocity, u, are constant across the cross-section of the jet, expressed the total current as the sum of the conduction and charge-transport currents,

$$I = \pi r^2 \sigma E + \rho \pi r^2 u q \qquad 1$$

where r is the jet radius, σ the conductivity of the liquid, and ρ the mass density. For the jet which disintegrates into small droplets, Hines developed a relation for the charge per unit mass of the droplets,

$$q_d = E_t\left[\frac{\epsilon^2\sigma}{2\pi\rho^2\dot{M}}\right]^{\frac{1}{3}} \qquad 2$$

where ϵ is the permittivity and E_t the electric field at the point where transition from conduction to charge transport occurs. Hines defined this transition point as being the point along the jet where the charge-transport current equals the conduction current. Multiplying Eq. 2 by $\dot{M}$ gives the current,

$$I = E_t\left[\frac{\epsilon^2\sigma\dot{M}^2}{2\pi\rho^2}\right]^{\frac{1}{3}} \qquad 3$$

Equation 3 gives a seemingly straight forward means of calculating the current for the jet; however the calculation of the electric field proves to be prohibitive due to the jet geometry involved. Hines gives a method by which this field can be calculated. This method, however, makes use of the value of current, along with measurements of jet dimensions taken from photographs, to find the field. Because this field calculation makes use of the current, this method of finding the field is not useful for predicting the current. Another difficulty with Eq. 3 is the assumption of constant conductivity. In low electric fields the conductivity of a liquid such as glycerine, $C_3H_5(OH)_3$, containing hydroxide groups is a result of electrolytic dissociation and impurities in the liquid. The conductivity is determined by the number of carriers present and their mobilities. For low electric fields the number of carriers for a pure liquid is constant with

$$C_a \cdot C_k = K \qquad 4$$

where C_a is the number of anions and C_k the number of cations per unit volume, and K the dissociation constant. Therefore the conductivity due to dissociation at low fields is constant. However for high fields, such as exist at the liquid surface,[7] additional carriers can be produced by increased dissociation and collision ionization.[12] Therefore, while the conductivity of the liquid within the jet can be considered constant, the conductivity along the jet surface is somewhat a function of the electric field strength at that surface.

Equation 3 seems to indicate that if the electric field varies directly with voltage, the current varies approximately directly with voltage, with some change due to conductivity variation. This expression can, however, be misleading since the location of the transition point changes with voltage changes. If the voltage were increased, the velocity of the jet would increase, and the transition to charge transport would occur closer to the capillary. The field decreases as the distance to the capillary decreases due to charge accumulation along the jet.[10] Therefore, the increase in the field due to the voltage increase would be offset somewhat by the decrease in field caused by the loca-shift of the transition point. As a result of this shift, current could be expected to vary at a rate lower than directly with voltage.

For the reasons outlined above, theoretical predictions of the current carried by the liquid jet proved to be too complex to be carried out. Therefore an empirical relation was sought so that trends in the current with changes of voltage, electrode spacing, and mass-flow rate could be determined and studied.

II. EXPERIMENTAL METHODS

All of the experimental work was conducted with the system shown in Fig. 3. By conducting the spraying in a vacuum of the order of 10^{-5} torr, corona effects were eliminated. Glycerine was used as the sprayed liquid; its relatively low vapor pressure was desirable in the vacuum. Spraying was accomplished by applying a potential of three thousand to eight thousand volts dc to the stainless steel capillary and brass plate. Glycerine was fed to the capillary at a pressure controlled by the height of the glycerine head in the vertical glass tube. This height was controlled by adjustment of the bellows by an exterior knob. Mass-flow rate was considered proportional to height as given by Poiseuille's equation,

$$\dot{M} = \frac{\pi R^4}{8\gamma L}(p_h - p_t) \qquad 5$$

where R is the inside radius of the capillary, γ the kinematic viscosity, L the capillary length, and p_h and p_t the pressure of the head and at the tip of the capillary respectively. The term p_t, because of the vacuum, could be due only to the electrical stress. Measurements indicated that the mass-flow rate was the same for no field as when spraying was taking place. Therefore for all measurements mass-flow rate was considered proportional to the column height.

Because of undesirable ripple in the power source, a bank of four 7.5 microfarad capacitors was used as the voltage source. A one-megohm resistor provided the necessary voltage drop to obtain traces on a differential input oscilloscope with one millivolt per centimeter sensitivity. In the single-jet or dc mode, current measurements were taken with a micro-micro ammeter. Some measurements were also taken with this meter in the multiple-jet region where the current measured was an average value.

Current measurements were taken in two sets, first by holding the height and voltage constant and varying the electrode spacing; second by holding height and electrode spacing constant and varying the voltage. The temperature was kept within one half a degree of 76 degrees Fahrenheit because of the large temperature dependence of the viscosity. Some measurements were also taken at 72 degrees.

Photographs were taken of the oscilloscope traces in the multiple-jet region. Photographs taken of the liquid spray were not made while data were being taken because of an apparent heating effect of the high intensity lamp.

III. EXPERIMENTAL RESULTS

Tables I and II contain experimental data obtained with the micro-micro ammeter in the spraying process at 76 degrees Fahrenheit. Table III contains current measurements made at 72 degrees Fahrenheit. Because of closer temperature control the data given in Table II are more consistent than the data of Table I; therefore the results from Table II were used in arriving at an empirical relation for current.

A. Current with a Single Jet

Figure 4 shows a family of curves obtained for current versus glycerine-column height (mass-flow rate) at a voltage of 4.0 KV. Plotting these values and those obtained at other voltage values on logarithmic paper showed straight lines with slopes varying from 0.51 to 0.58. The average slope was found to be about 0.55 so that current expressed in terms of column height gave

$$I \propto \dot{M}^{0.55} \qquad 6$$

Figure 5 shows a family of curves obtained for current versus voltage at a column height of 36 inches. Logarithmic plots of these values and others obtained showed relatively straight lines with slopes averaging about 0.45. Therefore

$$I \propto V^{0.45} \qquad 7$$

Figure 6 shows a family of curves obtained for current versus electrode spacing at a column height of 36 inches. Logarithmic plots of these values gave an average slope of about -0.18. Therefore

$$I \propto d^{-0.18} \tag{8}$$

where d is the spacing.

The total empirical expression found for current as a function of voltage, spacing, and height of glycerine column is

$$I = 0.57H^{0.55}V^{0.45}d^{-0.18} \tag{9}$$

The value of current was found to be highly temperature dependent. Figure 7 shows the current for both 76 and 72 degrees Fahrenheit at a column height of 36 inches. The current was found to decrease by from sixteen to eighteen per cent when the temperature was dropped from 76 degrees to 72 degrees.

An item of interest noted while data were being taken was that at a given height of glycerine, or mass-flow rate, as the voltage was decreased the minimum current at the point of jet breakup into the pulse mode was the same regardless of the spacing. Figure 8 shows the plot of these minimum currents.

B. Current with Jet Instability

With the electrode spacing from two to four millimeters there was observed an instability which greatly affected the current characteristic. In this region the position of the jet was influenced by the collected glycerine on the plate. Figure 9 shows a series of positions for the jet. In (a) the jet was in its normal position, with the collected glycerine below. Figure 9 (b) and (c) show the jet climbing because of the disturbed electric field. At the position shown in (d) the jet reached a maximum angle with the axis. At this point it was noted that the jet, previously solid, had broken up at the tip, with small droplets being formed. Also at the capillary tip there was a buildup of glycerine so that a large jet was formed as the cycle repeated (a).

Figure 10 (a) shows the oscilloscope trace of the current waveform exhibited by this unstable jet. Each oscillation represents a cycle of the rising jet, with each maximum corresponding to position (a) of Fig. 9. Figure 10 (b) shows the current waveform for the same spraying mechanism described above, but with the spacing increased to the critical point beyond which stable-jet spraying took place. Here the alternating jet reached a steady-state condition. Both the condition shown in Fig. 10 (a) and (b) were continuous waveforms.

C. Current with Multiple-Jet Spraying

Figure 11 shows the current measurements made with the micro-micro ammeter for a single jet, two jets, and three jets, along with average values for intermediate pulsing stages. The addition of a pulsing side spray to the single jet resulted in a pulse on top the dc level as shown in Fig. 12 (a). Increasing the electric field resulted in two constant jets which gave a constant current. The trace shown in Fig. 12 (b) is for the brush spray of Fig. 2 (c). The noise-like signal superimposed on the dc level was due to multiple-pulsing in addition to the numerous jets.

IV. DISCUSSION OF EXPERIMENTAL RESULTS

A. Current with a Single Jet

A comparison of the empirical equation, given here in general terms, and Eq. 3, repeated here for convenience,

$$I = E_t \left[\frac{\epsilon^2 \sigma \dot{M}^2}{2\pi\rho^2}\right]^{\frac{1}{3}} \qquad 3$$

$$I = K_I \dot{M}^{0.55} V^{0.45} d^{-0.18} \qquad 10$$

shows a certain agreement in the mass-flow rate relation, but indicates, as previously mentioned, that the actual current-voltage relation is far from being linear as indicated by Eq. 3, assuming the field to be proportional to the applied voltage. One possible explanation of the almost square root relation of voltage could come from the relation of acceleration to velocity of the liquid. Consider a point, x_1, along the jet near the capillary so that the current is primarily due to conduction. For low values of axial distance, x, the acceleration as a function of distance as given by Hines[10] can be approximated as

$$a = k_1 x^2 \qquad 11$$

From kinetic energy considerations it can be shown that at the point x_1 the velocity of the liquid is related to its acceleration as given by

$$u^2 = k_2 a \qquad 12$$

If acceleration is proportional to electric field, which is in turn proportional to voltage, and since

$$\dot{M} = \rho \pi r^2 u = \text{constant} \qquad 13$$

the conduction term for current from Eq. 1 becomes

$$I = k_3 \sqrt{V} \qquad 14$$

for constant mass-flow rate and spacing. The forgoing development is only a possible explanation because the assumptions made have not been validated; however this development does lend insight into jet-geometry effects.

Given the empirical relation for current versus mass-flow rate, and assuming the conductivity factor as given in Eq. 3 to be approximately correct, the current change with changes of temperature can be predicted.

$$I = k_4 \dot{M}^{0.55} \sigma^{0.33} \qquad 15$$

From Eq. 5,

$$M \alpha \frac{1}{\gamma} \qquad 16$$

and for a viscous liquid,[11]

$$\sigma \alpha \frac{1}{\gamma} \qquad 17$$

Therefore

$$I = k_5 \gamma^{-0.88} \qquad 18$$

Taking the logarithm and differentiating with respect to temperature gives, in finite difference form,

$$\frac{1}{I}\frac{\Delta I}{\Delta T} = -0.88\frac{1}{\gamma}\frac{\Delta \gamma}{\Delta T} \qquad 19$$

For glycerine in the range of temperature involved here, the percentage increase of viscosity per degree Fahrenheit is about five per cent.[13] Therefore, for a four degree decrease in temperature, an eighteen per cent decrease in current is expected as a result of viscosity increase. The observed current decrease was sixteen to eighteen per cent.

B. Current with Jet Instability

The jet instability described in Chapter III, Section B, was evidently caused by the presence of collected glycerine on the plate. A possible explanation of the jet's rising in the cycle could be the reduction of the velocity of the jet along the lower boundary due to the collection of glycerine there. This reduced velocity would cause a buildup of positive charge, repelling the rest of the jet upward, As the jet rose, the field along the axis would begin to draw glycerine along the axis. A buildup of glycerine near the capillary would result, reducing the size of the rising jet. The resulting small radius of curvature would increase the electric field stress until jet disintegration occurred. Then when the axial field at the glycerine buildup reached the critical value, a new jet would form along the axis. The oscillations in the current waveform would not be so much a function of the jet's oscillating position as it would be the oscillating cross section of the jet. The approximately exponentially rising envelope of Fig. 10 (a) was apparently caused by a gradual increase in the axial field.

C. Current with Multiple-Jet Spraying

The current for multiple-jet spraying as shown in Fig. 11 took distinct jumps as new jets were added. An approximate relation for two jets can be found by considering each of the jets to behave according to the empirical relation. Assuming the mass flow to be equally distributed, the relation for total current becomes,

$$I = 2K_I\left(\frac{\dot{M}}{2}\right)^{0.55} V^{0.45} d^{-0.18} \qquad 20$$

Similarly for three jets,

$$I = 3K_I\left(\frac{\dot{M}}{3}\right)^{0.55} V^{0.45} d^{-0.18} \qquad 21$$

As seen in Fig. 11, Eq. 20 is a fair approximation to the actual current, but the error with Eq. 21 is quite large.

V. CONCLUSIONS

The empirical relation determined for electrically-sprayed glycerine gives an accurate description of the current in the single-jet and, to a certain extent, the double-jet regions. Since no practical theoretical approach is known for predicting this current, an empirical relation would be required for other types of sprayed liquids. The use of high-speed photography to study the effects on jet geometry of changing the various parameters could give new insights leading to a more general description of the current.

The small magnitude of the current and inconvenience of the system would make the use of an electrical-spraying system to satisfy a current-voltage relation unlikely. The most probable use of the knowledge of this current would be in measurements of mass flow of an electrically-sprayed liquid by the use of a calibrated ammeter.

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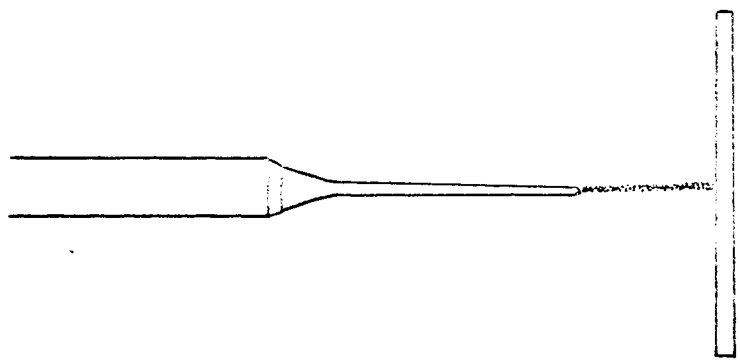


Fig. 1 Single jet with disintegrating tip.

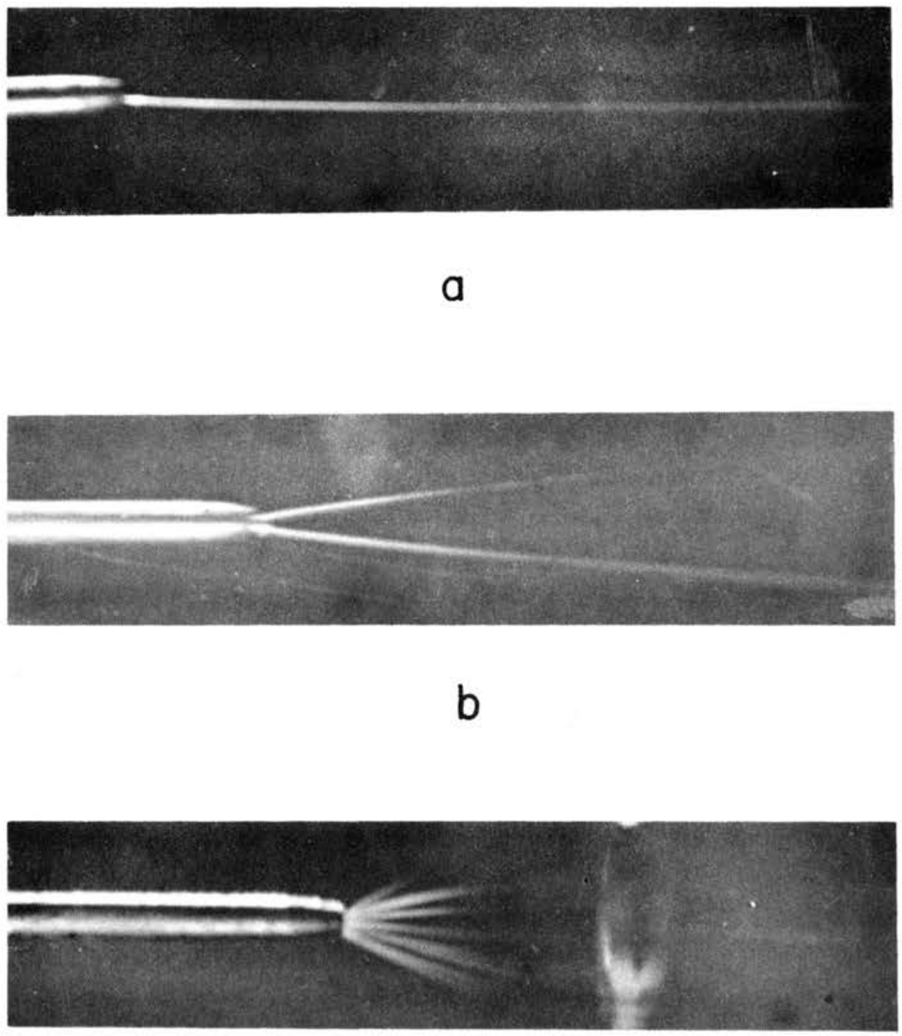


Fig. 2. Single and multiple jet spraying (a) single jet (b) double jet (c) multiple or "brush" spray

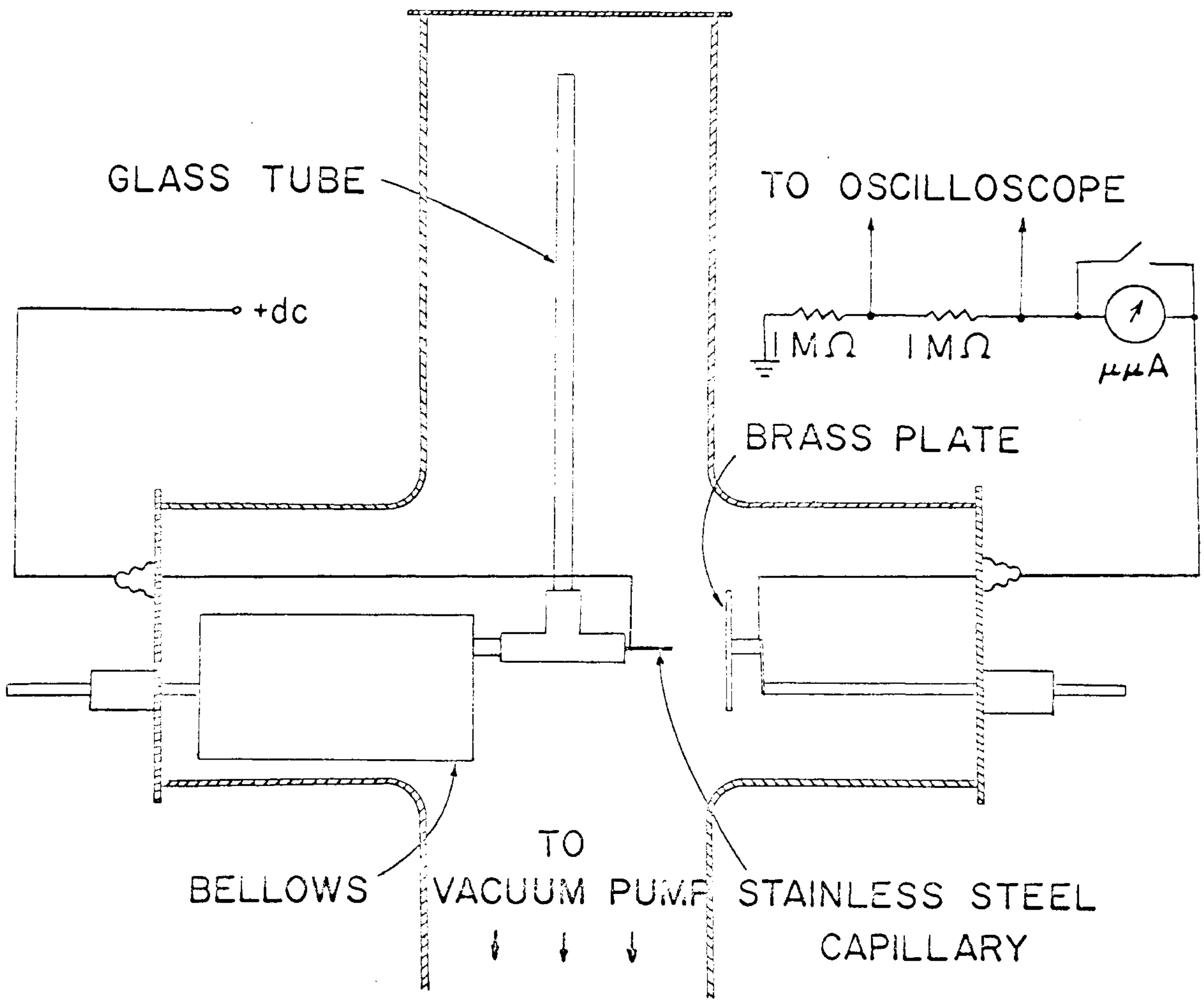


Fig. 3 Experimental equipment.

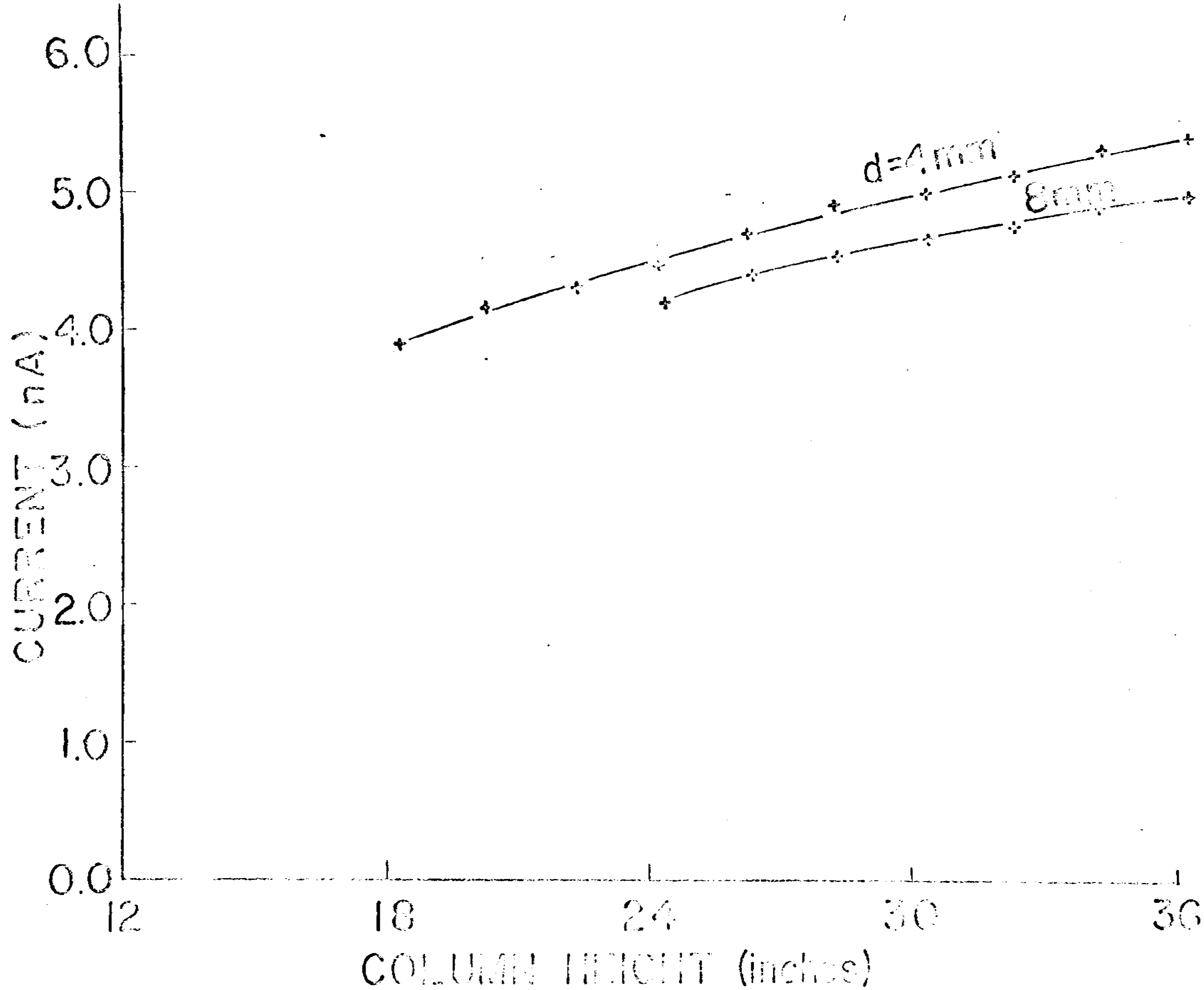


Fig. 4 Current versus column height, with voltage 4.0 KV.

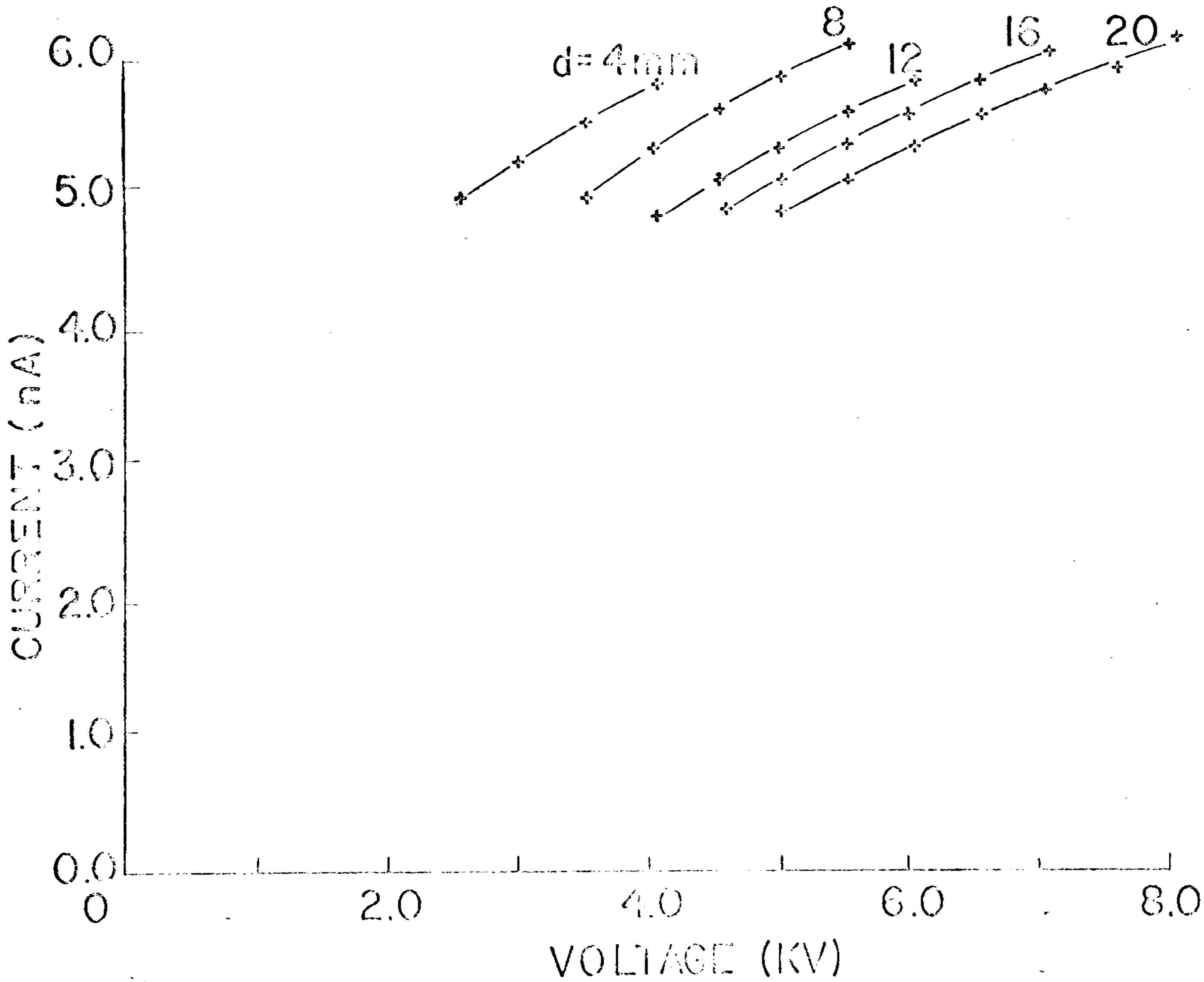


Fig. 5 Current versus voltage, with column height 36 inches.

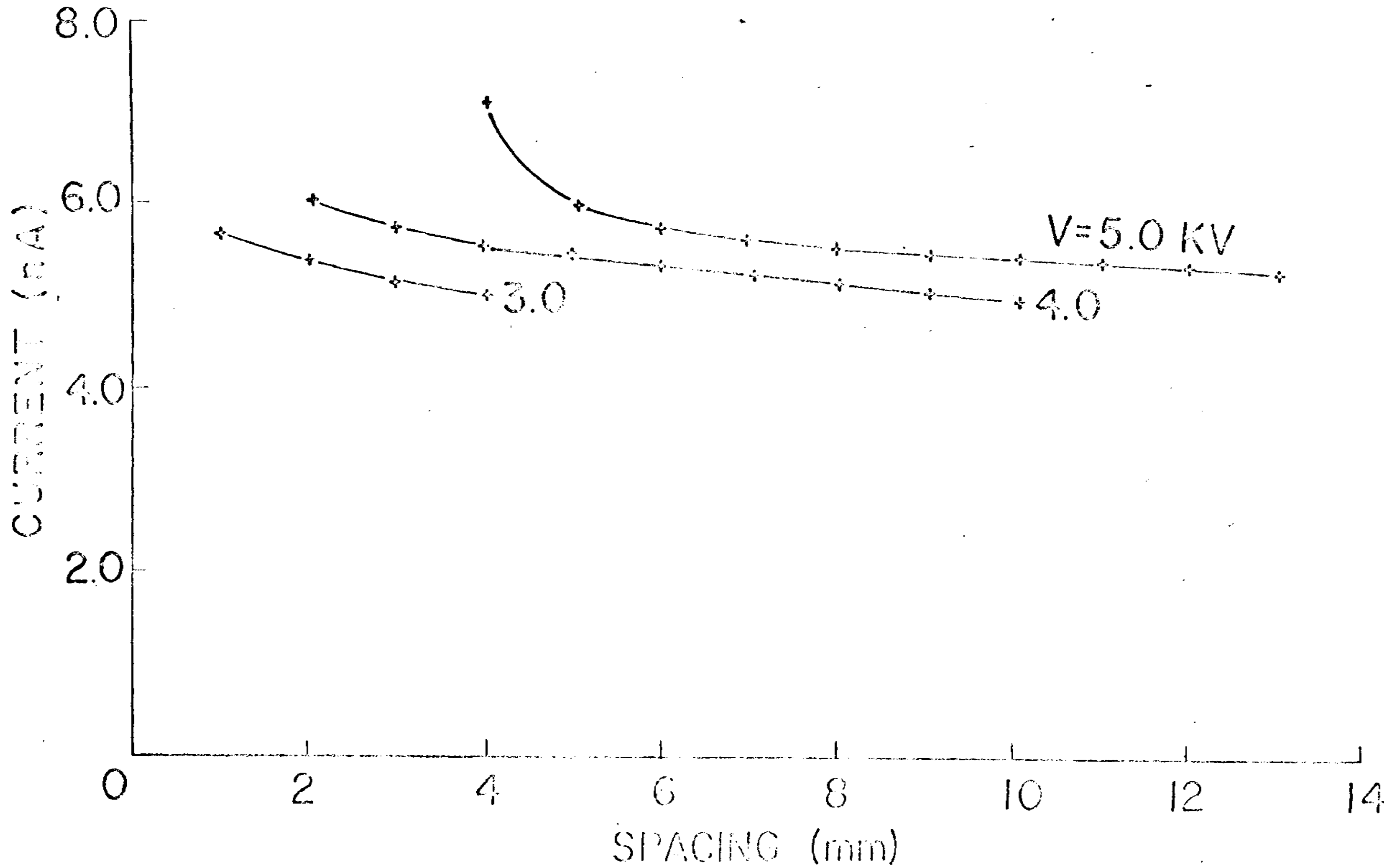


Fig. 6 Current versus electrode spacing, with column height 36 inches.

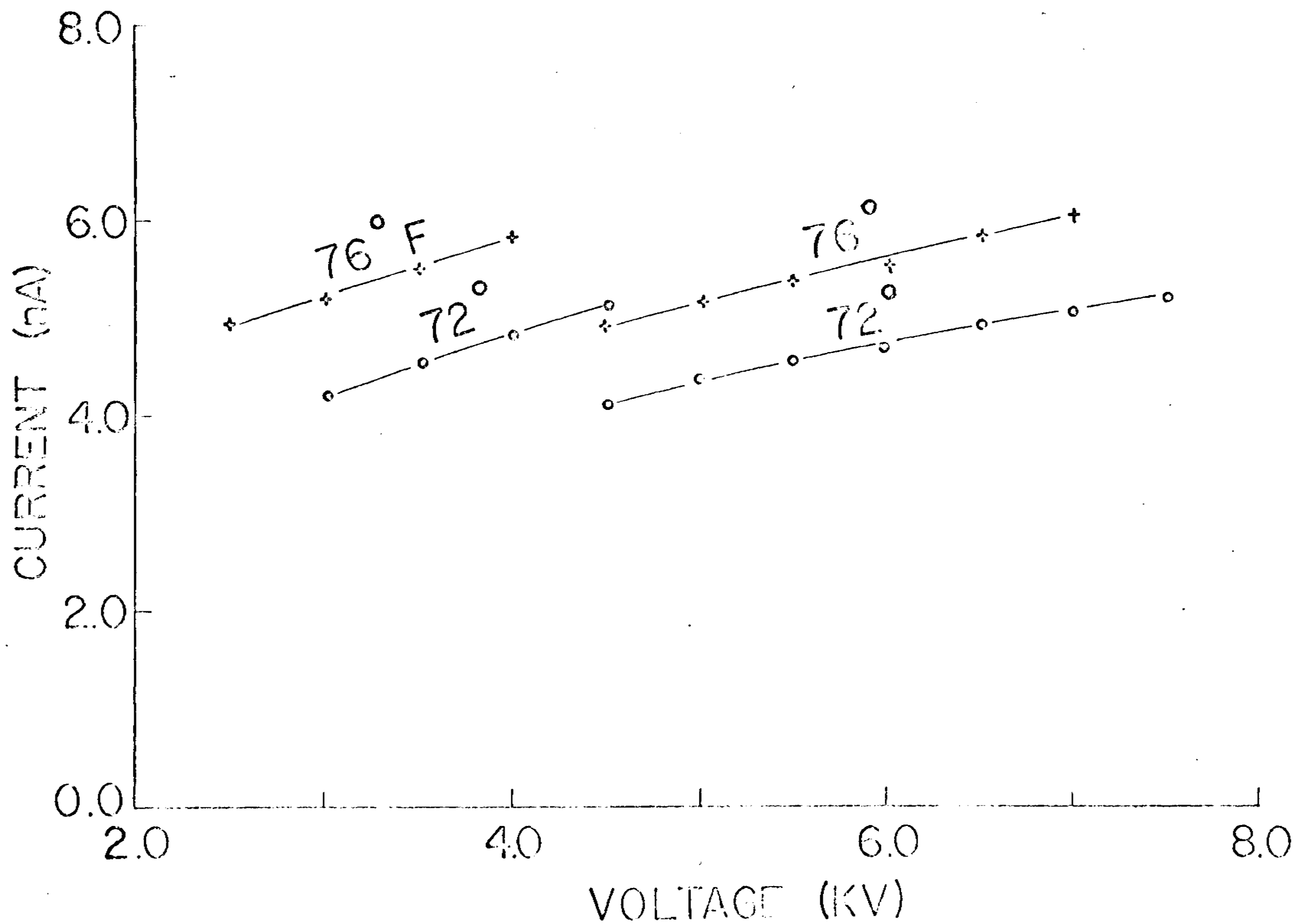


Fig. 7 Currents at 76 °F and 72 °F, with column height 36 inches.

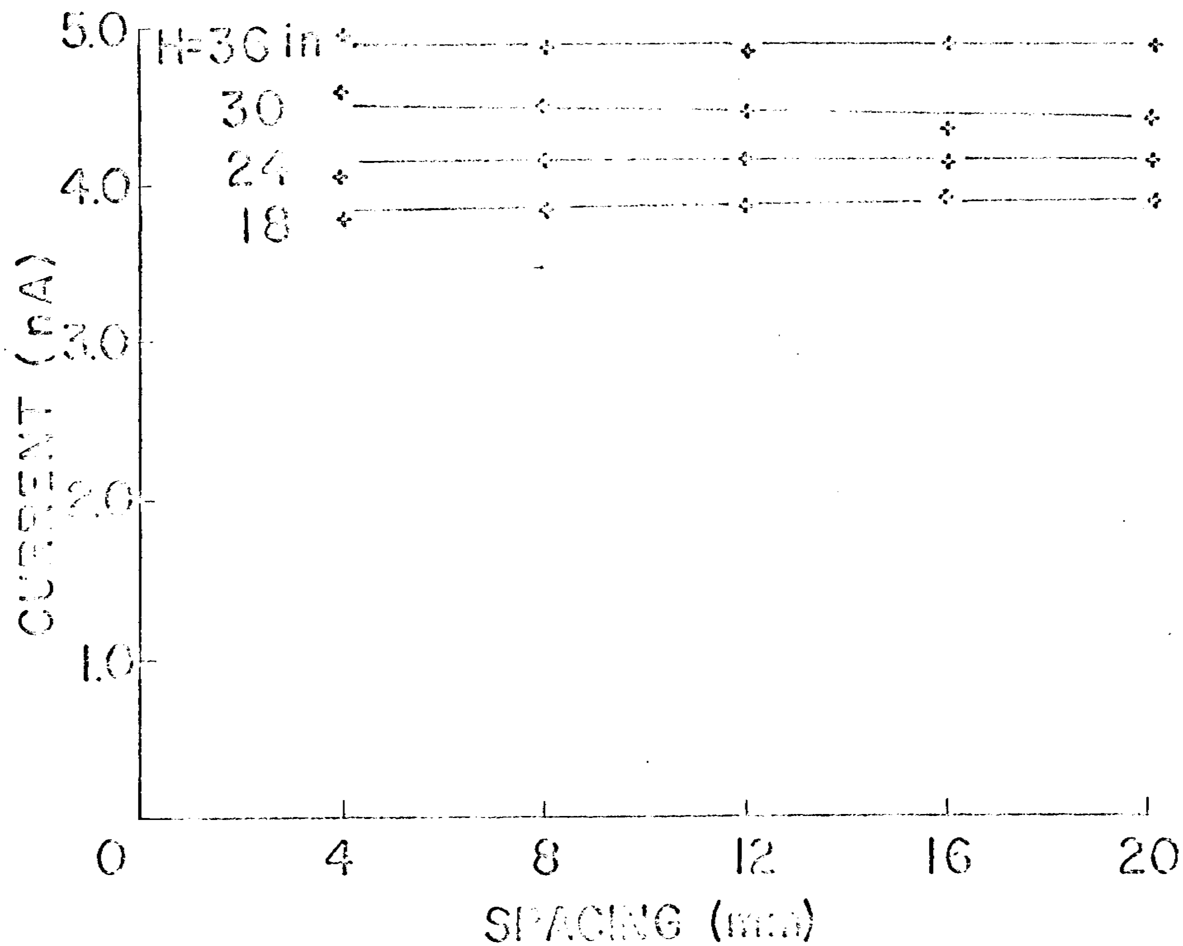


Fig. 8 Minimum currents before jet breakup into pulse mode.

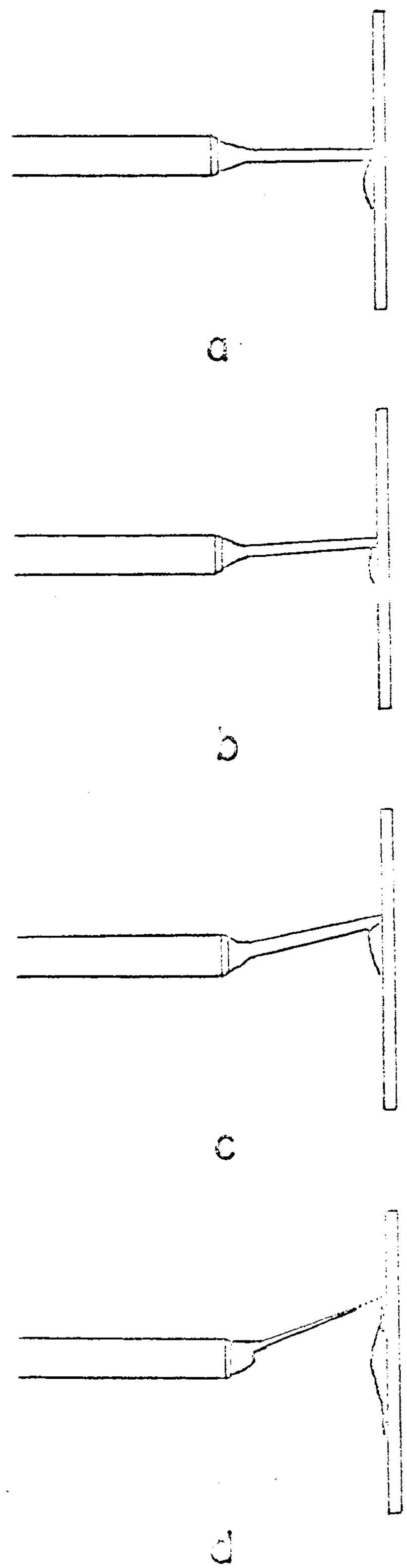
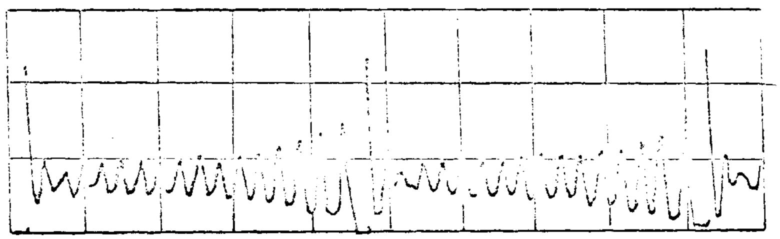
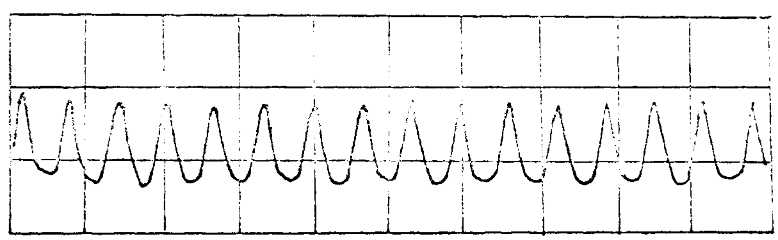


Fig. 9 Unstable jet in four positions of cycle.



a



b

Fig. 10 Oscilloscope trace for current with unstable jet, (a) "exponentially rising" (4 na/cm, 0.2 sec/cm), (b) steady state (4na,cm, 0.1 sec/cm).

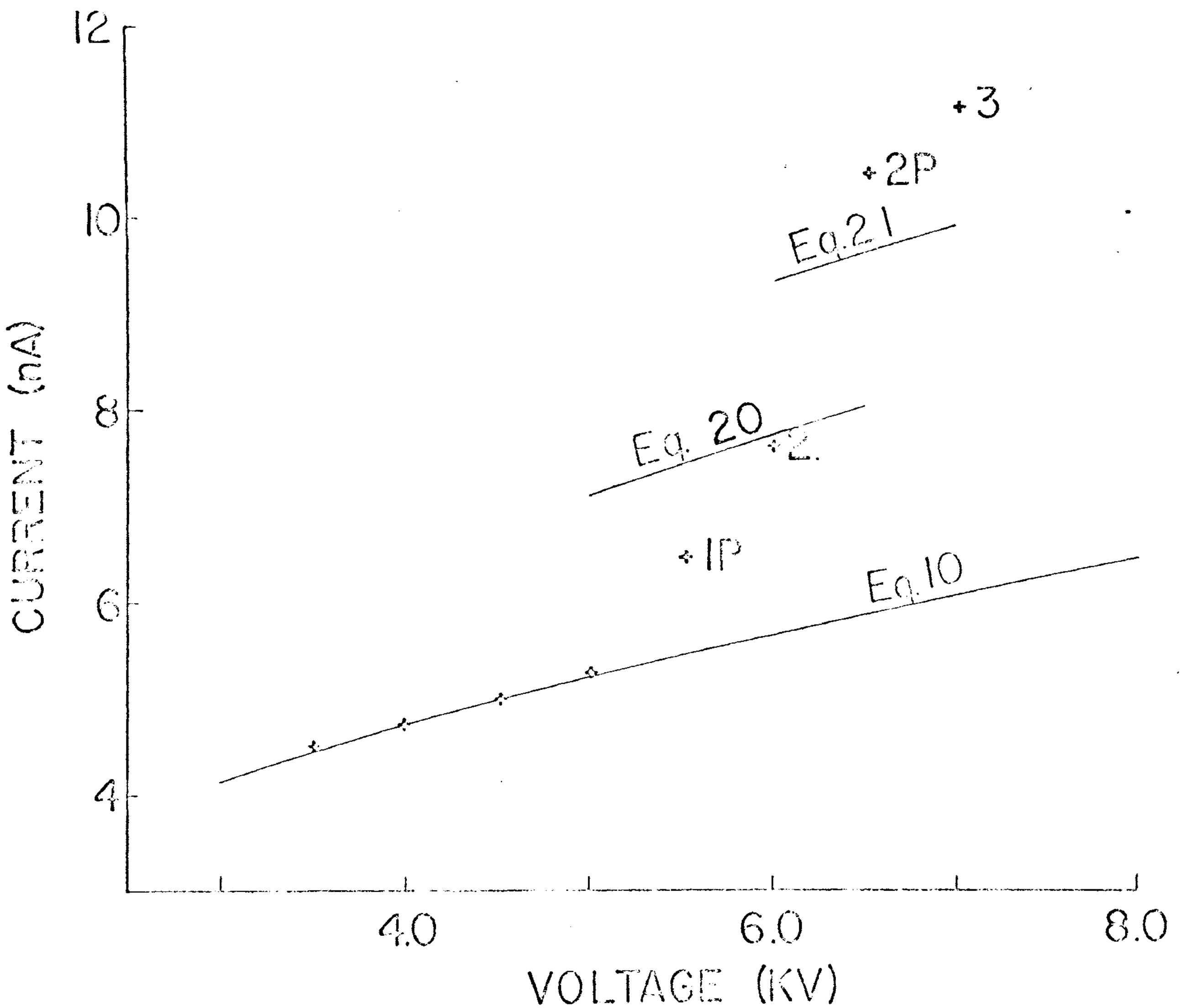
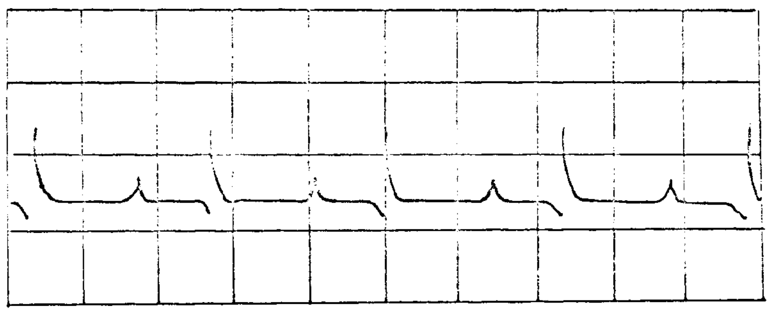
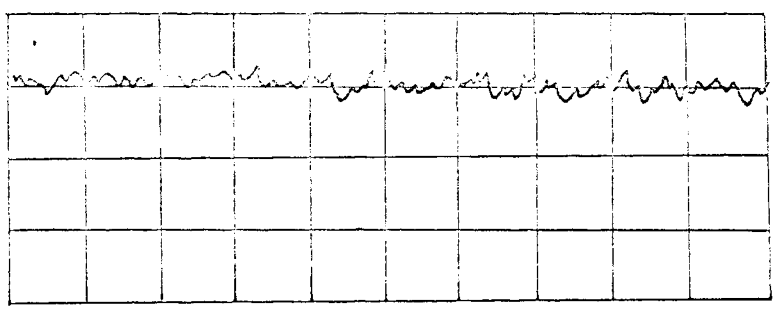


Fig. 1. Comparison of currents calculated by equations 10, 20, and 21, with currents measured for single jet, single jet plus pulse (1P) double jet (2), double jet plus pulse(2P), and triple jet (3).



a



b

Fig. 12 Oscilloscope trace of current, (a) single jet with pulse off capillary edge (4na/cm, 50 ms/cm), (b) multiple-jet or "brush" spray (4na/cm, 20 ms/cm).

Table I. Current for various column heights, voltages, and spacings at 76° F.

Voltage (KV)	Electrode spacing (mm)	18 in.	24 in.	30 in.	36 in.
5.0	3	5.60	6.55	7.20	8.20
5.0	4	4.05	4.80	5.30	7.10
5.0	5	3.95	4.65	5.10	5.90
5.0	6	3.90	4.65	5.00	5.70
5.0	7	3.85	4.45	4.85	5.60
5.0	8		4.35	4.75	5.50
5.0	9		4.30	4.70	5.40
5.0	10		4.25	4.65	5.35
5.0	11		4.15	4.55	5.30
5.0	12		4.10	4.50	5.20
5.0	13		4.05	4.45	5.15
5.0	14		4.00	4.35	5.05
5.0	15		3.95	4.30	4.95
5.0	16		3.90	4.25	4.90
5.0	17			4.20	4.85
5.0	18				4.80
4.0	2	3.85	4.45	5.00	5.60
4.0	3	3.75	4.30	4.75	5.40
4.0	4	3.65	4.15	4.60	5.30
4.0	5		4.10	4.50	5.15
4.0	6		4.05	4.40	5.05
4.0	7		3.95	4.30	4.95
4.0	8		3.90	4.25	4.90
4.0	9		3.85	4.15	4.80
4.0	10			4.10	4.70
4.0	11				4.60
3.0	1		4.50	5.15	5.65
3.0	2		4.35	4.85	5.35
3.0	3		4.10	4.70	5.15
3.0	4			4.45	4.95

Table II. Current for various column heights, voltages, and spacings at 76 °F.

Column height (in.)	Voltage (KV)	4 mm	8mm	12mm	16mm	20mm
18	8.0			8.80^{3}	6.10^{2}	5.30^{2}
18	7.5			8.40^{3P}	5.75^{2}	4.10
18	7.0	14.7^{N}	8.95^{3}	5.80^{2}	4.25	3.90
18	6.5	13.8^{N}	8.30^{3}	5.50^{2}	4.15	P
18	6.0	10.9^{3}	5.75^{2}	4.05	P	
18	5.5	8.70^{3P}	5.30^{2}	P		
18	5.0	6.90^{2}	3.95			
18	4.5	5.45^{2}	P			
18	4.0	3.95				
18	3.5	P				
24	8.0		13.5^{N}	9.80^{3}	6.75^{2}	5.90^{2P}
24	7.5		11.9^{N}	8.20^{3P}	6.40^{2P}	4.70
24	7.0		9.90^{3}	6.70^{2}	4.80	4.60
24	6.5		9.45^{3}	6.00^{2P}	4.65	4.45
24	6.0	11.2^{N}	6.70^{2}	4.75	4.50	4.30
24	5.5	9.55^{3}	5.80^{2P}	4.50	4.30	4.15
24	5.0	6.60^{2}	4.60	4.30	4.10	P
24	4.5	5.80^{2P}	4.40	4.15	P	
24	4.0	4.70	4.20	P		
24	3.5	4.30	P			
24	3.0	P				
30	8.0		15.2^{N}	11.5^{3}	8.05^{2}	5.45
30	7.5	18.8^{N}	13.4^{N}	10.8^{3}	7.25^{2P}	5.30
30	7.0	18.2^{N}	11.2^{3}	7.70^{2}	5.65	5.10
30	6.5	16.1^{N}	10.5^{3}	6.95^{2P}	5.45	4.95
30	6.0	13.0^{4}	7.65^{2}	5.40	5.25	4.85
30	5.5	11.0^{3}	6.45^{2P}	5.15	4.85	4.65
30	5.0	7.60^{2}	5.25	4.90	4.50	4.45
30	4.5	6.30^{2P}	5.00	4.70	4.35	P
30	4.0	5.15	4.75	4.45	P	
30	3.5	4.90	4.50	P		
30	3.0	4.60	P			
30	2.5	P				

Note: Superscripts indicate the number of jets, with N meaning numerous. No superscriptmeans one jet. Pulsing jets are denoted by superscript P. The P at the bottom of each column denotes the start of pulsing.

Table II. (Continued)

Column Height (in.)	Voltage (KV)	4mm	8mm	12mm	16mm	20mm
36	8.0		16.0^{N}	12.1^{3}	8.55^{2}	6.20
36	7.5		13.9^{3}	10.3^{3P}	7.70^{2P}	5.95
36	7.0		12.2^{3}	8.30^{2}	6.05	5.75
36	6.5	16.3^{N}	10.4^{3P}	7.40^{2P}	5.85	5.55
36	6.0	14.8^{N}	8.25^{2}	5.85	5.55	5.35
36						
36	5.5	12.1^{3}	6.10	5.60	5.35	5.10
36	5.0	8.65^{2}	5.80	5.30	5.15	4.85
36	4.5	7.10^{2P}	5.55	5.05	4.90	P
36	4.0	5.85	5.30	4.80	P	
36	3.5	5.50	4.95	P		
36	3.0	5.20	P			
36	2.5	P				

Table III. Current for various column heights, voltages, and spacings at 72 $^{\circ}$ F.

Column height (in.)	Voltage (KV)	4mm	8mm	12mm	16mm	20mm
30	8.0	17.9^{N}	13.0^{N}	9.20^{3}	6.40^{3P}	4.80
30	7.5	16.9^{N}	10.1^{4P}	7.75^{3P}	6.45^{2}	4.70
30	7.0	15.8^{N}	9.30^{3}	6.25^{2}	4.85	4.55
30	6.5	13.0^{N}	7.65^{3P}	5.60^{2P}	4.65	4.40
30	6.0	10.2^{4P}	6.20^{2}	4.60	4.50	4.25
30	5.5	8.80^{3}	4.80	4.45	4.35	4.10
30	5.0	6.20^{2}	4.50	4.25	4.05	3.95
30	4.5	4.80	4.30	4.05	3.90	P
30	4.0	4.50	4.10	3.90	P	
30	3.5	4.25	3.90	P		
30	3.0	4.00	P			
30	2.5	P				
36	8.0		13.8^{N}	9.80^{3}	6.35^{2P}	5.20
36	7.5		11.4^{4P}	6.75^{2}	5.25	5.10
36	7.0	16.0^{N}	9.60^{3}	6.25^{2P}	5.05	4.95
36	6.5	13.8^{N}	7.10^{2}	5.20	4.90	4.80
36	6.0	10.5^{4P}	6.60^{2P}	4.95	4.70	4.65
36	5.5	8.65^{3}	5.20	4.75	4.55	4.50
36	5.0	6.70^{2}	5.00	4.50	4.40	4.25
36	4.5	5.10	4.75	4.30	4.15	P
36	4.0	4.80	4.45	4.05	P	
36	3.5	4.55	4.15	P		
36	3.0	4.25	P			
36	2.5	P				

VITA

Larry Eugene Stoddard was born April 6, 1945, at Mobridge, South Dakota. His primary and secondary education was received at Humboldt, South Dakota, where he graduated from high school in May, 1963. The following September he enrolled at South Dakota State University at Brookings, where he received his Bachelor of Science Degree in Electrical Engineering in June 1967.

He enrolled in the Graduate School of the University of Missouri--Rolla in September 1967, and held a teaching assistantship until June, 1968. In September, 1968, he became an NDEA Graduate Fellow.